Time: 3 Hours **MATHS TEST 8** Max. Marks: 100

General Instructions:

(1) *All questions are compulsory.*

(2) *The Question Paper consists of 29 questions divided into 3 Sections A, B and C. Section A comprises of ten questions of 1 mark each, Section B comprises of twelve questions of 4 marks each and Section C comprises of seven questions of 6 marks each*

(3) *All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.*

(4) *There is no overall choice. However, internal choice has been provided in four question of 4 marks each, two questions of 6 marks each You have to attempt only one of the alternatives in all such questions.*

(5) *Use of calculators is not permitted. However, you may ask for mathematical tables.*

SECTION – A

Q1. Show that $*: R \times R \to R$ given by $(a,b) \to a + 4b^2$ is a binary operation.

Q2. If **a** and **b** are unit vectors making an angle θ with each other then find $|\vec{a} - \vec{b}|$

Q3. Evaluate : $\int (\tan x + \cot x)^2 \, dx$

Q4. Write $\cot^{-1}\left(\frac{1}{\sqrt{x^2 - 1}}\right), |x| > 1$ in the simplest form.

Q5. Solve the equation for x, y, z and t, if $2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$

Q6. Evaluate : $\int \frac{\sin(x-a)}{\sin x} dx$

Q7. Let A be a non-singular square matrix of order 3×3. Find |adj A| .

Q8. Find the direction cosines of the vector $3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$.

Q9. Evaluate : $\int \frac{1}{1 + e^x} dx$

Q10. If a matrix has 24 elements, what are the possible orders it can have ? What if it has 13 elements ?

SECTION – B

Q11. Evaluate : $\int \frac{1 + 2x^2}{x^2\left(1 + x^2\right)} dx$

Q12. Show that the area of the triangle formed by the tangent and the normal at the point (a, a) on the curve $y^2(2a - x) = x^3$ and the line x = 2a is $\frac{5a^2}{4}$.

Q13. Prove using the properties of determinants :

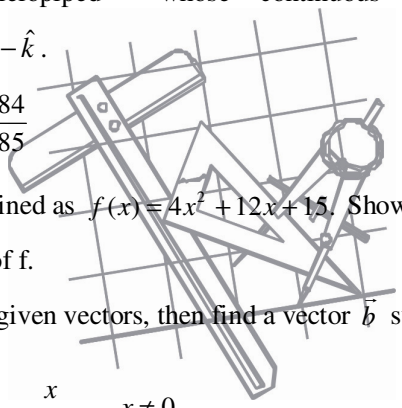
$$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (a+c)^2 & b^2 & ca \\ (b+a)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)\left(a^2+b^2+c^2\right)$$

Q14. A and B throw a die alternatively till one of the them gets a '1' and wins the game. Find their respective probabilities of winning, if B starts first.

Q15. If $x^p y^q = (x+y)^{p+q}$, find $\frac{dy}{dx}$.

Q16. Evaluate : $\int \frac{x^2-1}{x^4+x^2+1}dx$

Q17. Find the volume of parallelopiped whose continuous edges are represented by vectors $2\hat{i}-3\hat{j}+\hat{k}, \hat{i}-\hat{j}+2\hat{k}$ *and* $2\hat{i}+\hat{j}-\hat{k}$.



Q18. Prove : $\sin^{-1}\frac{3}{5}-\sin^{-1}\frac{8}{17}=\cos^{-1}\frac{84}{85}$

Q19. Let $f: N \to R$ be a function defined as $f(x)=4x^2+12x+15$. Show that $f: N \to S$, where S is the range of f, is invertible. Find the inverse of f.

Q20. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{c}=\hat{j}-\hat{k}$ are given vectors, then find a vector $\vec{b}$ such that $\vec{a}\times\vec{b}=\vec{c}$ and $\vec{a}.\vec{b}=3$.

Q21. Prove that the function $f(x)=\begin{cases} \frac{x}{|x|+2x^2}, & x\neq 0 \\ k & , x=0 \end{cases}$ is continuous at x= 0 regardless the value of k.

Q22. Using limit of sums, evaluate : $\int_1^4 \left(x^2-x\right)dx$

SECTION – C

Q23. A toy company manufactures two types of dolls, A and B. Market tests and available resources have indicated that the combined production level should not exceed 1200 dolls per week and the demand for dolls of type B is at most half of that for dolls of type A. Further, the production level of dolls of type A can exceed three times the production of dolls of other type by at most 600 units. If the company makes profit of Rs 12 and Rs 16 per doll respectively on dolls A and B, how many of each should be produced weekly in order to maximize the profit ?

Q24. Find the area of region $\{(x, y): 0 \le y \le x^2 + 3, 0 \le y \le 2x + 3, 0 \le x \le 3\}$.

Q25. A window is in the form of a rectangle above which there is a semi circle. If the perimeter of the window is p cm, find the radius of semi circle for maximum light entered through the window.

Q26. Use the product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations :

$x - y + 2z = 1; 2y - 3z = 1; 3x - 2y + 4z = 2$

Q27. Find the shortest distance and the vector equation of the line of the shortest distance between the lines given by $\vec{r} = (-\hat{i} + 5\hat{j}) + \lambda(-\hat{i} + \hat{j} + \hat{k})$ and $\vec{r} = (-\hat{i} - 3\hat{j} + 2\hat{k}) + \mu(3\hat{i} + 2\hat{j} + \hat{k})$.

Q28. A pack of playing cards was found to contain 51 cards. If the first 13 cards which are examined are all red, what is the probability that the missing card is black.

Q29. (a) Solve the differential equation : $\frac{dy}{dx} = \frac{x^2 y}{x^3 + y^3}$

(b) Solve the differential equation : $\left(1 + e^{\frac{x}{y}}\right)dx + \left(1 - \frac{x}{y}\right)e^{\frac{x}{y}}dy = 0$